



Description Logic

Summer Semester 2022

Exercise Sheet 6

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Exercise 6.1 We consider TBoxes $\mathcal{T}$ that only contain the following two kinds of axioms:

- role inclusions of the form $r \sqsubseteq s$, and
- role disjointness constraints of the form $\text{disj}(r, s)$,

where r and s are role names. An interpretation $\mathcal{I}$ satisfies these axioms if

- $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$, and
- $r^{\mathcal{I}} \cap s^{\mathcal{I}} = \emptyset$, respectively.

Modify the tableau algorithm $\text{consistent}(\mathcal{A})$ to decide consistency of $(\mathcal{T}, \mathcal{A})$, where $\mathcal{A}$ is an ABox and $\mathcal{T}$ a TBox that contains only role inclusions and role disjointness constraints. Show that the algorithm remains terminating, sound, and complete.

Exercise 6.2 Let $\mathcal{K} = (\mathcal{T}, \mathcal{A}_0)$ be an $\mathcal{ALC}$ -knowledge base, where $\mathcal{T}$ is a general TBox. A *precompletion* of $\mathcal{K}$ is a clash-free ABox $\mathcal{A}$ that is obtained from $\mathcal{K}$ by exhaustively applying all expansion rules except the $\exists$ -rule.

Show that $\mathcal{K}$ is consistent iff there is a precompletion $\mathcal{A}$ of $\mathcal{K}$ such that, for all individual names a occurring in $\mathcal{A}$, the concept $C_{\mathcal{A}}^a := \bigcap_{a: C \in \mathcal{A}} C$ is satisfiable w.r.t. $\mathcal{T}$.

Exercise 6.3 Let C be an $\mathcal{ALC}$ -concept. We denote by $\#C$ the number of occurrences of the constructors $\sqcup$, $\sqcap$, $\exists$, and $\forall$ within C . The multiset $M(C)$ contains, for each occurrence of a subconcept of the form $\neg D$ in C , the number $\#D$.

Use this representation to prove that exhaustively applying the following transformation rules to an $\mathcal{ALC}$ -concept always terminates, regardless of the order of rule applications:

$$\neg(C \sqcap D) \rightsquigarrow \neg\neg\neg C \sqcup \neg\neg\neg D$$

$$\neg(C \sqcup D) \rightsquigarrow \neg\neg\neg C \sqcap \neg\neg\neg D$$

$$\neg\neg C \rightsquigarrow C$$

$$\neg(\exists r.C) \rightsquigarrow \forall r.\neg C$$

$$\neg(\forall r.C) \rightsquigarrow \exists r.\neg C$$